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# CORRECTION NOTE TYPICAL CONFIGURATION FOR ONE-DIMENSIONAL RANDOM FIELD KAC MODEL[1]

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Estimate (3.39) which appears in the proof of Proposition 3.4 in [*Ann. Probab.* **27** (1999) 1414–1467] is wrong. We present below a corrected proof which introduces an extra factor 2 in equations (3.34) and (3.35). This has no consequence in the rest of the paper since Proposition 3.4 is used to estimate only ratios; see (3.23) and (3.25).

In Proposition 3.4 in [1], the condition $m \in \{-1, -1+2/|B|, -1+4/|B|, \dots, 1-2/|B|, 1\}$ has to be added. This is harmless since Proposition 3.4 is used for proving Proposition 3.1, where this assumption is done. Moreover, (3.34) and (3.35) must be replaced respectively by

$$\Psi_{z,\alpha,m} = \frac{2}{\sqrt{2\pi|B|}\sigma_z}\left(1 \pm \frac{66}{|B|\sigma_z^2}\right) \tag{1.1}$$

and

$$\Psi_{z,\alpha,m} = \frac{2}{\sqrt{2\pi|B|}\sigma_z}\left(1 \pm \frac{66}{g(|B|)}\right). \tag{1.2}$$

Below we outline the arguments to get (1.2), the case of (1.1) is similar.

In the proof of Proposition 3.4, inequality (3.39) is clearly wrong for $k = \pm\pi$. Since, for $y \in [0,1]$, we have $|ye^{-2ik} + (1-y)|^2 = 1 - 2y(1-y)(1-\cos(2k))$ and $1 - s \le e^{-s}$ for all $s \in \mathbb{R}$, it is easy to see that

$$\left|\frac{\cosh(x \pm ik)}{\cosh(x)}\right| \le \exp\left[-\frac{1-\cos(2k)}{4\cosh^2 x}\right] \tag{1.3}$$

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that replaces (3.39). Then, using $\cos(x) \le 1 - \frac{x^2}{2} + \frac{x^4}{4!}$, it can be checked that, for $k \in [0, \pi]$,

$$1 - \cos(2k) \ge 2\left(1 - \frac{\pi^2}{12}\right)(k^2 \wedge (k-\pi)^2), \tag{1.4}$$

from which one gets, for $k \in [0, \pi]$,

$$|\Phi(z, \alpha, k)| \le \exp\left[-\frac{(1-\pi^2/12)(k^2 \wedge (k-\pi)^2)}{2}|B|\sigma_z^2\right], \tag{1.5}$$

where $\Phi(z, \alpha, k)$ is defined in (3.38) and $\sigma_z$ is defined in (3.28) in [1]. Formula (1.5) replaces (3.40) in [1]. As a consequence, (3.41) has to be replaced by

$$\widetilde{\mathcal{E}}_\rho = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathbb{1}_{\{\rho<|k|\le\pi-\rho\}}\Phi(z, \alpha, -k)e^{ikm|B|}\, dk. \tag{1.6}$$

Then choosing as in [1], $\rho = (\sigma_z\sqrt{|B|})^{-1} f(|B|)$ with

$$f(|B|) = \sqrt{\frac{2}{1-\pi^2/12}\log g(|B|)}, \tag{1.7}$$

where $g$ is as in Proposition 3.4 in [1], one gets

$$|\widetilde{\mathcal{E}}_\rho| \le \frac{1}{\sqrt{2\pi|B|}\sigma_z}\left(\frac{2}{\sqrt{\pi(1-\pi^2/12)\log g(|B|)}}\right)\frac{1}{g(|B|)}, \tag{1.8}$$

that replaces (3.48) in [1]. Calling as in [1] [see (3.45)],

$$\Psi_{z,\alpha,m}(\rho) = \frac{1}{2\pi}\int_{-\rho}^{+\rho} e^{ik|B|m}\Phi(z, \alpha, k)\, dk, \tag{1.9}$$

introducing the two quantities

$$\begin{aligned} I_2 &= \frac{1}{2\pi}\int_{-\pi}^{-\pi+\rho} e^{ik|B|m}\Phi(z, \alpha, k)\, dk, \\ I_3 &= \frac{1}{2\pi}\int_{\pi-\rho}^{\pi} e^{ik|B|m}\Phi(z, \alpha, k)\, dk. \end{aligned} \tag{1.10}$$

After simple algebra, using that $m = -1 + \frac{2l}{|B|}$ for some $l \in \mathbb{Z}$ and elementary change of variables, one gets the crucial relation

$$I_2 + I_3 = \Psi_{z,\alpha,m}(\rho). \tag{1.11}$$

Now $\Psi_{z,\alpha,m}$ defined in (3.37) satisfies

$$\Psi_{z,\alpha,m} = 2\Psi_{z,\alpha,m}(\rho) + \widetilde{\mathcal{E}}_\rho. \tag{1.12}$$

The extra factor 2 we mention in the abstract is the one in (1.12). Using the same computations done after (3.45) in [1], one gets (1.2).

## REFERENCE

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